

Social and socio-mathematical norms in collaborative problem-solving

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Based on the notions of social and socio-mathematical norms we investigate how these are established during the interactions of pre-service teachers who solve mathematical problems. Norms identified in relevant studies are found in our case too; moreover, we have found norms related to particular aspects of the problems posed. Our results show that most of these norms, once established, enhance the problem-solving process. However, exceptions do exist, but they have a local orientation and a relatively small influence.

En s'appuyant sur les concepts des normes sociales et 'socio-mathématiques', nous avons étudié comment ces normes se sont établies au cours des interactions entre les enseignants et les étudiants en activité de résolution de problèmes mathématiques. Aux résultats de la recherche apparaissent d'une part les mêmes normes qui ont été déjà remarquées à d'autres recherches relatives et d'autre part des normes liées plus particulièrement aux problèmes posés. Les résultats de la recherche montrent que dans la majorité des cas les normes aident le processus de la résolution des problèmes. Il existe bien sûr des exceptions, mais elles ont une influence et une orientation locale.

Basierend auf den Begriffen der sozialen und sozio-mathematischen Normen untersuchen wir, wie diese in die Interaktionen von angehenden Lehrern beim Lösen von mathematischen Problemen einfließen. Normen, welche in relevanten Studien identifiziert werden, wurden in unserem Fall ebenfalls gefunden. Wir haben ausserdem Normen gefunden, welche sich auf bestimmte Aspekte der Fragestellungen beziehen. Unsere Resultate zeigen, dass die meisten Normen, sind sie einmal etabliert, die Problemlösungsprozess verbessern. Es bestehen zwar Ausnahmen, doch diese haben eine lokale Orientierung und einen relativ kleinen Einfluss.

Basados en las nociones de la norma social y sociomatemática, estamos investigando cómo se establecen dichas normas durante las interacciones de los profesores en pre-servicio, que resuelven problemas matemáticos. Las normas identificadas en estudios relevantes también se encuentran en nuestro caso; de hecho, hemos hallado normas relacionadas con aspectos particulares de los problemas expuestos. Nuestros resultados demuestran que la mayor parte de estas normas, una vez establecidas, mejoran el proceso de solución de problemas. Sin embargo, existen también excepciones pero éstas tienen una orientación local y una relativamente menor influencia.

Keywords: social norms; sociomathematical norms; collaborative problem solving; symbolic interactionism; mathematics

The interactions contained in mathematics teaching are a rich field for analysis; contemporary researchers draw their attention to various aspects of them in order to clarify issues related to mathematical learning. On the one hand, we find studies that

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focus on the way mathematical meanings are interactively created and established. This is usually done by a linguistic content analysis of the discussions taking place. Sfard (2001) uses a network flowchart to demonstrate how meanings are gradually constructed by the speakers. Pimm (1987) examines the connection between mathematical and everyday language. This is similar to the work of Pirie (1998) and Moschkovich (2003). On the other hand, we find studies that incorporate the social-psychological factor in their analysis. This is done by examining the social context surrounding the interaction (Dekker, Wood, and Elshout-Mohr 2001), the various rules (and meta-rules) that regulate the interaction (Yackel and Cobb 1996; Sfard 2000; Yackel 2001) or by interpreting the participants' acts from a certain social or psychological perspective (Mercer 1995; Rowland 2000).

Most of the studies mentioned examine teacher-student interactions or students' interactions with the teacher holding a scaffolding role. The participants in the majority of these studies are students in primary or high school, with the exception of Yackel (2001) and Rowland (2000). This fact led us to form a study about pre-service teachers cooperating in pairs with no scaffolding taking place. From the participants' point of view we aimed to:

- a) provide them with experience in cooperative problem-solving, a method which they may have to implement during their teaching;
- b) demonstrate to them that problem-solving does not necessarily require sophisticated formulas and processes;
- c) give them a good opportunity to refresh their existing mathematical knowledge and acquire new skills related to problem-solving.

From our point of view we aimed to:

- a) identify the norms established in the particular setting;
- b) compare these norms with those found in relative research;
- c) examine their effect in the solving process.

This paper will deal only with the second focus of our study, i.e. the one related to our own intentions. For this purpose, we shall initially present our theoretical framework, namely symbolic interactionism enhanced with elements of conversation analysis.

Theoretical framework

The sociological theory of symbolic interactionism (Mead 1934; Blumer 1969) is based on the premise that human interactions are made possible through the use of various linguistic and non-linguistic symbols. When a person enters a particular setting (e.g. a room where he is about to solve a mathematical problem in collaboration with somebody else) firstly he defines the situation in which he is involved. This is made possible from existing information about the setting, the participants and the possible (or desired) outcomes of the interaction. For example, when a student enters a problem-solving session, he might (or might not) have some information concerning the other participants; whether the session will be audio- or video-taped; what kind of problems he will have to solve, etc. While the interaction proceeds, all participants jointly produce a definition of the situation, i.e. a mutual agreement as to whose claims concerning what issues will be temporarily honoured

(Goffman 1971, 21). In other words, the participants agree on a common set of *prescriptions*. This agreement assists the participants in their attempt to *interpret* the symbols that may come up during the encounter. In the case of collaborative problem-solving mathematical symbols such as expressions, formulas or figures contribute to the interactive formation of mathematical meanings.

The nature and the function of the prescriptions that guide social interactions has been the focus of a number of sociological studies. Biddle and Thomas (1966) define prescriptions as 'behaviours that indicate that other behaviours should (or ought to) be engaged in. Prescriptions may be specified further as demands or norms, depending upon whether they are overt or covert, respectively' (103). For example, each culture possesses certain rules (i.e. demands) concerning behaviour in social interactions, from the clothes one is expected to wear to the way one addresses the other. Norms, on the other hand, are covert in nature; Homans (1966) describes them as ideas in the minds of people, ideas which 'can be put in the form of a statement specifying what the members [of a group] or other men should do, ought to do, are expected to do, under given circumstances' (134). For example, a student is not supposed to interrupt the teacher's talk; on the contrary, the teacher may interrupt the student in order to correct him or to maintain the class order.

While doing mathematics, students and teachers adhere to similar rules and norms; particularly, they adhere to general rules and norms that apply to every social interaction, like the norm concerning the interruptions we have already mentioned. Moreover, participants adhere to a set of *socio-mathematical norms*, i.e. 'normative aspects of mathematics discussions specific to students' mathematical activity' (Yackel and Cobb 1996, 461). Some examples of socio-mathematical norms are the understandings of what counts as mathematically different, sophisticated, efficient and elegant (Yackel 2001). Whether these norms can be a subject of teaching or some sort of guidance is an important issue; Yackel and Cobb (1996) speak of classes following the 'inquiry tradition' (462), where the teacher plays the role of the representative of the mathematical community. Sfard (2000) – who uses the term 'meta-discursive rule' – argues that:

... the unique rules of mathematical discourse can neither be learned by a simple articulation, nor can they be re-invented by students engaged into discussing mathematical problems 'in any way they regard as appropriate'. Rules of language games can only be learned by actually playing the game with more experienced players. (185)

What happens then, when two adult students are asked to solve a mathematical problem without any guidance from a mathematician? That question led us to focus our research on interactions of university students with no scaffolding taking place. But the theory of symbolic interactionism together with the notion of norms were not enough for the analysis we intended; that is why we incorporated conversation analysis in our framework. Conversation analysis examines the utterances produced in a particular *context*; more than that, context is not treated as a static property of the situation, but as an entity that is continuously re-shaped by the on-going discourse (Heritage 1984). The scope of the researcher is to locate the relevant (for his purpose) patterns in discourse in order to examine features such as turn-taking or concept creation. This is particularly useful in mathematical talk, where concepts and procedures are continuously introduced and negotiated.

Methodology

The participants in our research were 40 undergraduate students of the Department of Primary Education (i.e. pre-service teachers) of the University of Ioannina in Greece. The students participated voluntarily in the study. They were asked to choose their partner in order to form 20 pairs. Thirteen pairs consisted of two females, six pairs consisted of a female and a male and one pair consisted of two males. Three one-hour sessions were held for each pair and one problem was assigned at a session. The sessions were held in a laboratory setting with only the two students and the observer (the first author) present. The only instructions given to the students were that they should verbalise every thought they make and that they should try to cooperate to solve the problems posed. The students were aware that the sessions were tape-recorded by the observer, whose interventions were the fewest possible (e.g. he did not reply to questions like 'Is this the right thing to do?', but he did give information concerning the remaining time or the geometrical instruments available). The time interval between the sessions of each pair varied between four to seven days. Once the dialogues were transcribed into written text we performed a two-level analysis (Lemke 1989; Mercer 1995). In the first level, i.e. the thematic analysis, we looked at the way mathematical concepts were created and negotiated during the interactions. The process of *thematic system development* (Lemke 1989) consists of patterns of talk found in the text; these patterns include adjacency pairs (i.e. two turns where the first establishes the 'conditional relevance' of the second, like question-answer) or larger conversational units. Our intention was to trace the mathematical concepts and procedures from the moment they were introduced to the moment they were used in the solution process.

In the second level, i.e. the interactional analysis, which is the focus of this paper, we initially looked at the way language was used by the participants to convey attitudes related to particular social and socio-mathematical norms.¹ Norms were 'inferred by identifying regularities in patterns of social interaction' (Yackel and Cobb 1996, 460). Then, the effect of these norms was examined by analysing the thematic patterns in each case. The usual pattern one expects to find is introduction-discussion-approval/disapproval; whenever there were changes or significant delays in that pattern, we looked for connections with particular norms that were established. This leads to the conclusion that the two levels were not in fact separated in our analysis; what we did was observe *how the norms established influenced the thematic patterns*. The analysis section that follows, will clarify this procedure.

Sample analysis

We will present examples from two pairs, in order to demonstrate our analytic procedure. All discussions are translated from Greek by the first author.

First pair

Paula and Joanna are the pseudonyms of two 22-year-old female students. Like all the pairs in the study they knew each other prior to the study. The following excerpt comes from their first session, when they were given the 'T-shirt problem'.

The design [in Figure 1] is going to be used on a T-shirt. You accidentally took the original design home, and your friend, Chris, needs it tonight. Chris has no fax

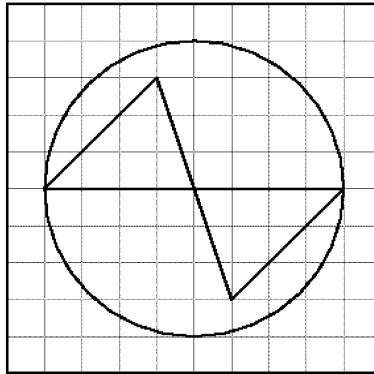


Figure 1. The T-shirt problem.

machine, but has a 10 by 10 grid just like yours. You must call Chris on the telephone and tell him precisely how to draw the design on his grid. Prepare for the phone call by writing out your directions clearly, ready to read over the telephone.

In the part preceding the dialogue below ([1]–[34]), the two students have agreed on the instructions related to the drawing of the circle and its diameter.

- 35 Joanna: These are a problem. [She refers to the triangles in the figure] He should draw a straight line again...
- 36 Paula: Where from?
- 37 Joanna: From the point where the diameter touches the circle...
- 38 Paula: Yeah, but how will he know where from, on what point? Look, if we tell him from here...
- 39 Joanna: And?
- 40 Paula: Tell him... in the square [inaudible] purely practically, describe the squares to him, count the squares and tell him go to the particular square and draw a line...
- 41 Joanna: You mean to put numbers in the small squares?
- 42 Paula: Yeah, but this is totally practical, it's not mathematical...

At the beginning of the excerpt, the concept of a straight line is introduced by Joanna; this concept belongs to the broader concept of the triangle included in the figure. To be precise, Joanna refers to a line segment, a concept which is understood by Paula, who identifies a basic property of it, i.e. its starting point. Joanna's reply in [37] is not explicit enough for Paula, who reacts in [38]. Her reaction reveals the following norms:

- the social norm that if you disagree with someone's opinion you are expected to justify your view;
- the social norm that in order to achieve a smooth cooperation and to avoid tension you are expected to express your disagreement in an indirect way (this is revealed by the 'Yeah' at the beginning of Paula's utterance);
- the social norm of cooperation; the students are expected to work together (this is revealed by the first person plural in 'we tell him');
- the socio-mathematical norm that a mathematical proposition is expected to be unambiguous.² This norm is related to the 'mathematical efficiency' norm found by Yackel and Cobb (1996);
- the socio-mathematical norm that a mathematical method is expected to be understood by a third person who reads it (in our case, Chris). This norm is

included in the non-ambiguity norm, but we separate it because it played an important role in the particular problem-solving process: the students strive to express unambiguous propositions in order to be understood by Chris.

In her next turn [40] Paula articulates her suggestion: she proposes the use of a system of coordinates. All the verbs she uses are in the first person plural,³ a fact that goes in line with the ‘cooperation norm’ we have mentioned. Paula’s suggestion reveals a socio-mathematical norm too: that there is a border between mathematics and general practice (however this may be conceived by the speakers). In [40] and [42] Paula stresses the fact that her process is ‘totally practical’ and ‘not mathematical’. This fact leads her to the abandonment of her suggestion after a few turns:

- 52 Paula: Eh, a line which will be at... You tell him, to count the squares. OK OK, he won’t get it, yes, right. Cause we can see it now.

The next part of the session ([53]–[137]) consists of the students’ continuous attempts to generate a set of clear instructions for the drawing of the two triangles. They reach a dead end because their expressions lack precision. Then, Joanna re-introduces Paula’s suggestion on the use of a coordinates system.

- 138 Joanna: What do we call these? The ones we... diagrams, not diagrams, what do we call them?
 139 Paula: You mean in mathematics, this...
 140 Joanna: Yeah, the one we drew a horizontal and a... a horizontal and a vertical... the ones we called x and y?

Finally, the two students adopt the use of a coordinates system, but the fact remains that they have wasted a considerable amount of time because of their common attitude towards the border between mathematics and practice.

Second pair

Tania and Sofia are the pseudonyms of two 21-year-old female students. The following excerpt comes from their second session, when they were given the ‘triangle problem’.

Figure 2 shows a triangle in which three lines are drawn to one or the other of the opposing sides from each of two vertices. This divides the triangle into 16 non-overlapping sections. If 14 lines are drawn in the same way, how many non-overlapping sections will the triangle have?

In the beginning of the session ([1]–[30]) Tania and Sofia misunderstood the problem; they thought that they were asked to find the number of line segments which produce 14 non-overlapping sections. After resolving this issue, they agreed

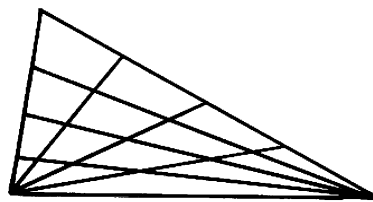


Figure 2. The triangle problem.

that drawing 14 segments is too difficult and time-consuming, so they started formulating a joint- method based on analogies; this is the point when the following dialogue took place:

- 77 Tania: We should do it that way, see how many we get and how else we could do it.
78 Sofia: Yeah, and then we see, then we'll see it. Let's do it that way. So, shall we do it with stepping on the unit? Yeah, let's do it with stepping on the unit.
79 Tania: How would it help you?
80 Sofia: Come on, let's get it over with!
81 Tania: No, we shan't do anything stupid. What's your rationale? I mean...
82 Sofia: To get a result.

Sofia then, by the use of analogies (three line segments produce 16 sections, therefore 14 line segments produce x sections) gets the result of 74.66... sections.

- 88 Sofia: That's the number of sections.
89 Tania: Is this possible? Does this make sense?
90 Sofia: That's the outcome!
91 Tania: I can see that.
92 Sofia: What did you expect?
93 Tania: It doesn't make sense. 74.6 doesn't make sense. It would either be a section or not.
94 Sofia: What can I do? That's the outcome.
95 Tania: So we must do something else. This doesn't make sense.

Unlike the previous example, the theme under discussion now is not a concept and its properties, but a procedure. This fact leads to the establishment of a different set of norms related to how a mathematical method may be validated. Tania in [77] introduces the idea that a mathematical method may be validated by its outcome; this idea is immediately accepted (so it becomes a socio-mathematical norm) by Sofia in [78], who goes on describing the method she intends to implement. Another norm striving to be established is related to the justification of a method, or by paraphrasing Tania's words in [79] how would a method help you. Sofia resists to the establishment of that norm and provides a weak justification for the analogies method: 'to get a result' [82]. In this case the participants have different understandings of what constitutes a 'proper' justification for a mathematical method. Maybe that is the reason for the tension we see especially in Sofia's speech ([80], [90] and [94]). This fact stresses the importance of establishing a set of norms, i.e. shared understandings, while cooperating to solve a problem. By implementing the analogies method Sofia gets a decimal number of sections; this is the point when another socio-mathematical norm appears: the result of an analytic procedure (or even of a single operation) is expected to 'fit' in the context of the problem. In our case, we need to have a whole number of sections, a fact stressed by Tania and, finally acknowledged by Sofia in [94]. We may note that the 'cooperation norm' found in the previous pair is also present here: all verbs in [77], [78], [81] and [95] are in the first person plural. But there are certain points when one participant (in our case Sofia) does not adhere to it; this leads to moments of tension as we have already mentioned. Finally, Tania's speech reveals her attitude towards expressing her opinion in a non-offensive way ([79], [89]), a norm we have found in the previous pair too.

The above analysis demonstrates that an understanding of participant (no matter if it becomes shared or not) may hinder or slow down the solution process. If the collaboration norm is absent, i.e. if the participants do not jointly resolve their

disputes, the process becomes problematic. One may argue that trial and error is an accepted method of problem-solving. Our point is that when two adult students are discussing a mathematical method, they are expected to have certain criteria to validate it.

From the thematic analysis point of view we may note that analogies play an important part in students' thematic repertoire: almost half of the pairs implemented a relative method before proceeding to another solution.

Results

The focus of our study is the social and socio-mathematical norms established in collaborative problem-solving and their effect in the solving process. Firstly, we shall present all the norms we have identified through the analysis of the transcribed discussions; the first three are social norms and the rest are socio-mathematical norms.

- a) Collaboration norm: the participants are expected to reach a mutual agreement on the solution process and its features, i.e. the concepts and procedures included. This is expressed through the first person plural of the verbs and the questions about the partner's opinion before implementing a method.
- b) Justification norm: one has to justify her opinion, especially when she expresses disagreement with her partner. This is expressed through words such as 'because' or 'that's why' in a sentence.
- c) Avoidance of threat norm: one is expected not to impose a threat towards her partner, i.e. not to insult her. This is expressed through indirect speech acts (Austin 1962) or by various politeness strategies (Brown and Levinson 1987).
- d) Non-ambiguity norm: mathematical expressions are expected to be clear and unambiguous. This is expressed through prompts for rephrasing.
- e) Third person comprehension norm: mathematical expressions are expected to be explicit enough so they can be understood by a third person that reads them. This norm, as we have already noted, is related to the non-ambiguity norm and is expressed through prompts for rephrasing, enhanced with references to the third person.
- f) Mathematical justification norm: mathematical methods need some sort of justification before their implementation; there needs to be a rationale to support their use. This is mainly expressed through questions beginning with 'why', e.g. 'Why should we use that method?'
- g) Mathematical differentiation norm: mathematical areas such as algebra and geometry are distinct, non-overlapping areas; there is also a differentiation between mathematical and everyday practices. This norm is expressed by the students' understandings of what counts as a 'mathematical' and a 'practical' solution to a problem. In the 'Triangle problem' it takes the form of differentiating between the 'geometrical' solution (i.e. draw 14 line segments) and the 'algebraic' one (i.e. find a formula that produces the number of the sections).
- h) Validation norm: mathematical methods need to be validated before and/or after they are implemented; this norm is related to the justification norm

according to the following scheme: introduction of a method-justification or method-validation of a method. A method may be validated by its difficulty, its time duration or even its result. In some cases the method may be validated by its 'identity', i.e. whether it is a 'pure mathematical' or a 'practical' method (see the differentiation norm above). The validation norm is expressed through queries for information on the above matters, or through relevant quotes, such as: 'Forget about it, it'll take us ages to make that' (taken from a pair discussing whether they should draw 14 line segments in the 'Triangle problem').

- i) Relevance norm: the outcome of a method is expected to be relevant to the problem's conditions, In other words, the result has to make sense. This is related to the validation norm, since an irrelevant result may probably lead to the withdrawal of a method. This norm is revealed by expressions like Tania's in our second example.

From the thematic development point of view, it is easy to see that norms influence the process of establishing a mathematical concept or method. Figure 3 aims at demonstrating the points when each norm, once established, may take part in the solution process. Note that the collaboration norm plays the most vital role, since it is involved in all the stages of thematic development.

The influence of most of the above norms has been positive as far as the concept development is concerned. Indeed, as the sample dialogues have shown, students have jointly established norms related to smooth cooperation and mathematical justification. This resulted in the normal flow of mathematical symbols during the interactions. The absence of the 'collaboration norm', even in small parts of the discussions, led to moments of tension and disorder; it was as if there were two independent people in the setting instead of a pair working collaboratively. Problems in the process were also created by the mathematical differentiation norm. The students were sometimes caught between that norm and the validation norm. In other words, they validated a method not by its difficulty or its efficiency, but by its 'mathematical' or 'non-mathematical' character. This led them to delays in the solving process.

Conclusion

The results of our study lead us to two different sets of conclusions. The first set concerns the nature of the students' interactions and the resulting implications for mathematics educators. The second set concerns the students' mathematical background that has led them to the establishment of particular norms. The fact that the participants in our study are adults who have known each other before the time of the study has definitely influenced the establishment of stable and smooth cooperation between them. The presence of the observer has also played an important role. Our results show that adult students with no history in collaborative problem-solving find it quite easy to establish the necessary social and socio-mathematical norms. This could be a prompt for a wider implementation of such methods in tertiary education. At this point we have to stress that we do not suggest the implementation of joint problem-solving with no guidance on behalf of the educators. Even in our case of adult participants there were instances when some guidance could have taken place (e.g. when the students were struggling to decide on

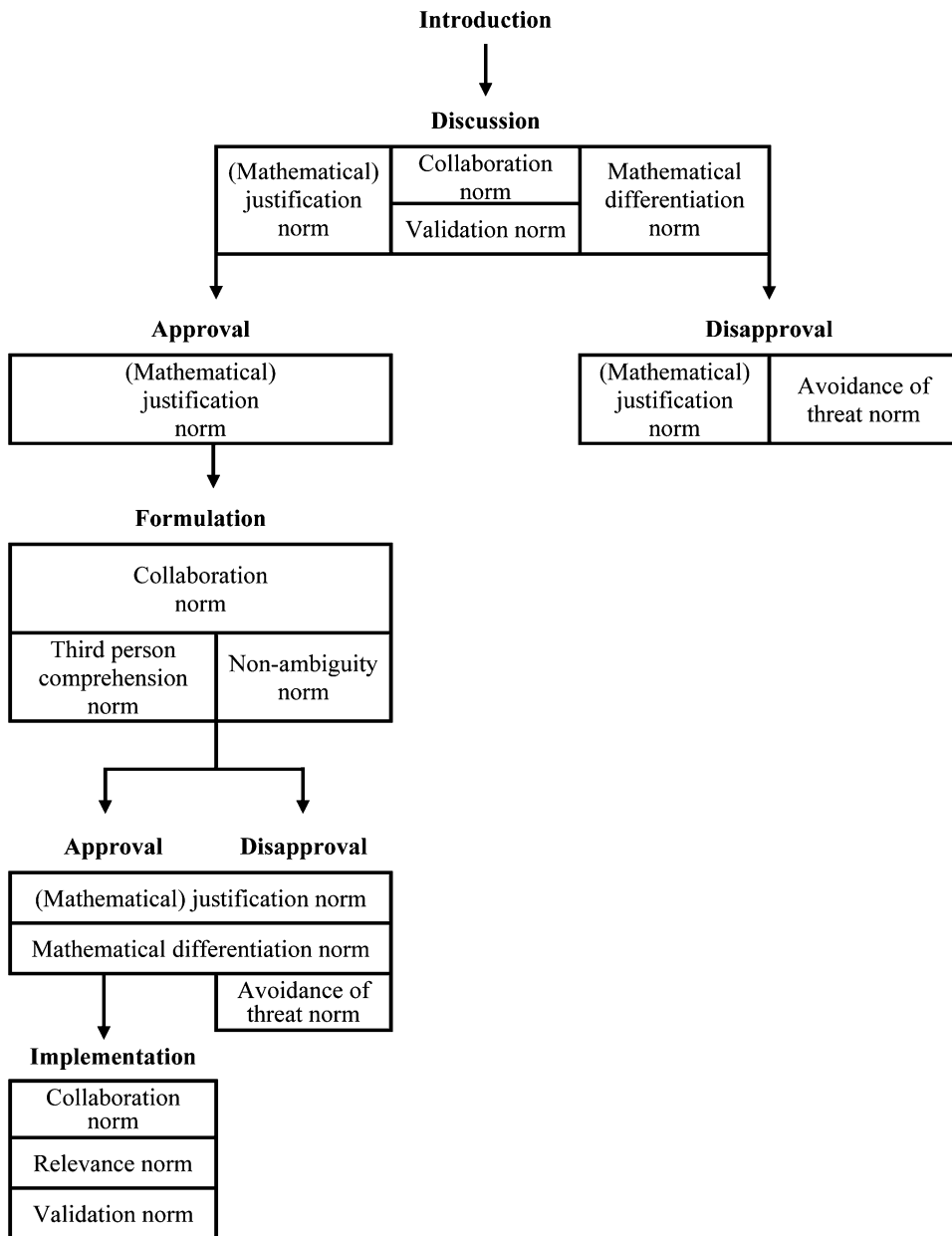


Figure 3. Norms and thematic development.

the criteria to validate a proposed method or when they could not resolve a dispute). The fact still remains that students can solve problems by cooperating and by sharing same views (norms) on how they should interact.

An interesting outcome of our study is that students hold specific views towards mathematics and its usefulness, especially when it comes to implementing a method. Moreover they seem reluctant to abandon these views. Since our case study is

context-dependent this conclusion holds for Greek students. Thus it seems that mathematics education in Greece has established some norms which sometimes hinder problem-solving. According to these norms, mathematics is comprised of distinct and non-related areas, such as algebra and geometry. Moreover, mathematical language is seen to be comprised mainly of formal terminology, where everyday language has no place. This fact needs to be further investigated in Greece and other countries.

Notes

1. This is not to say that language is not used to express other attitudes too, like those related to personal needs. A basic personal need is to save one's own 'face', i.e. to protect oneself against disapproval.
2. The fact that mathematical propositions are expected to be unambiguous does not necessarily lead to the view that ambiguity should be avoided by all means (Rowland 2000).
3. Greek verbs have different endings according to the person, so one does not need a pronoun to identify it.

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